# A POSTERIORI ERROR ESTIMATION AND SUPERCONVERGENCE WITH DISCONTINUOUS GALERKIN METHODS FOR HYPERBOLIC CONSERVATION LAWS

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We develop *A posteriori* error estimates for hyperbolic problems that are based on a superconvergence property where (smooth) discontinuous Galerkin solutions converge at a higher rate at the outflow boundaries of element than they do globally. In particular, we [1] show that DG solutions of one-dimensional hyperbolic conservation laws using piecewise-polynomials of degree $p$ have a strong $O(h^{2p+1})$ convergence rate at the outflow ends of elements and a higher $O(h^{p+2})$ convergence rate at the roots of the Radau polynomial of degree $p+1$ on elements than their global $O(h^{p+1})$ rate. We use this result to construct asymptotically correct spatial discretization error estimates for DG solutions of conservation laws.

Similar results hold for multi-dimensional problems. While Radau polynomials are not defined on unstructured triangular meshes, we develop a theory in terms of a Dubiner basis of orthogonal polynomials on triangular elements. The key argument in the development is a demonstration that integrals of the discretization error vanish to leading order on each element and, simultaneously, on the outflow edges of each element. With this, an *a posteriori* estimate of the discretization error is obtained as a linear combination of Dubiner polynomials of degrees $p$ and $p+1$ on each element [2]. Since Radau polynomials are a combination of (the orthogonal) Legendre polynomials of degrees $p$ and $p+1$, our results may be regarded as their extension to two dimensions. We demonstrate superconvergence by showing that the average local discretization error on outflow edge(s) of elements converges as $O(h^{2p+1})$. Thus, discretization errors propagate between elements at a high order, and we use this to obtain global discretization error estimates. Examples indicate that these *a posteriori* error estimates are generally within 10% of the actual errors for wide ranges of mesh spacings and polynomial degrees.

## References

[1] S. Adjerid, K.D. Devine, J.E. Flaherty, and L. Krivodonova, "A posteriori error estimation for discontinuous Galerkin solutions of hyperbolic problems," *Comp. Meths. Appl. Mech. Engng.*, v. 191, p. 1997–1112, 2002.

[2] L. Krivodonova and J.E. Flaherty, "Error estimation for discontinuous Galerkin solutions of multidimensional hyperbolic problems," *Adv. in Comput. Maths.*, to appear, 2003.